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LETTER TO THE EDITOR

Non-uniform long-range order in certain random systems

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Abstract. We consider a special class of random anisotropic spin models, characterised by macroscopically preferred directions for spin alignment. Spin wave arguments applied to these models predict that ferromagnetism is unstable below four dimensions. We show that unlike the case of random axes distributed uniformly over the entire unit sphere, this spin-wave result is spurious and ferromagnetism can indeed exist below four dimensions. The ordering is very non-uniform, thus explaining why spin wave theory which is based on the assumption of uniform order yields instabilities.

Much attention has been focused recently on the properties of vector spin models with quenched random site impurities, in particular, the value of the lower critical dimensionality below which long-range ferromagnetic order does not exist. Imry and Ma (1975) showed by a very elegant domain argument that a random field will destroy long-range order below four dimensions for n-vector models with $n \geq 2$. Using a variety of arguments, Pelcovits *et al* (1978) arrived at a similar conclusion for the case of random uniaxial anisotropy. This result was generalised by Aharony (1981) to include p-fold random anisotropies, with uniaxial anisotropy corresponding to $p = 2$. In all of these models, the local easy directions for spin ordering vary randomly over the entire surface of the n-dimensional unit sphere. The strength of the anisotropy may or may not be random.

Aharony (1978) has further generalised these results to an n-vector spin system with the following Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - \sum_{i,j} \sum_{\alpha\beta=1}^{n} \Delta_{ij}^{\alpha\beta} S_i^\alpha S_j^\beta. \qquad (1)$$

The spin $\boldsymbol{S}_i$ at site i has n components indexed by α. The first term in (1) describes a non-random, nearest-neighbour exchange term, while the coefficients $\Delta_{ij}^{\alpha\beta}$ are random. However, the configurational averages of the latter coefficients maintain the n-dimensional spin space isotropy. Aharony claimed that any type of random off-diagonal coefficients $\Delta_{ij}^{\alpha\beta}$ (i.e., $\alpha \neq \beta$) will destroy the ferromagnetic long-range order below four dimensions. This result was obtained by assuming that an ordered phase exists with finite magnetisation in, e.g., the 1-direction and then shifting S^1 by M. This shift generates a term

$$\Delta H = \sum_j \boldsymbol{H}_j \cdot \boldsymbol{S}_j \qquad (2)$$

where

$$H_j^\alpha = M \sum_i \Delta_{ij}^{1\alpha}. \qquad (3)$$

The field $\boldsymbol{H}_j$ is thus a local random field. Consideration of the spin wave fluctuations due to the transverse component of $\boldsymbol{H}_j$ shows that these fluctuations diverge at and below four dimensions. Hence one concludes that $M = 0$ for $d \leqslant 4$.

In this note, we show that the above result of Aharony is *not* true in general. While the transverse spin fluctuations may diverge in a spin wave approximation, this divergence only rules out *uniform* long-range ferromagnetic order. Indeed, we will now consider a subclass of models described by (1) where the deviation in spin alignment from site to site can be as great as 90°, yet long-range order still exists.

Specifically, we consider $\Delta_{ij}^{\alpha\beta} = \Delta_i^{\alpha\beta}\delta_{ij}$, i.e., site randomness. The $n = \infty$ version of this model with off-diagonal randomness has been studied by Hertz *et al* (1982). At each site we rotate the spin coordinate system to diagonalise the second term in (1). We then obtain, in place of (1),

$$H = -\sum_{\langle ij\rangle} \sum_{\alpha,\beta\gamma=1}^{n} JR_i^{\alpha\beta} R_j^{\alpha\gamma} \tilde{S}_i^{\beta} \tilde{S}_j^{\gamma} - \sum_i \sum_{\alpha=1}^{n} \lambda_i^{\alpha} (\tilde{S}_i^{\alpha})^2 \tag{4}$$

where $R_i^{\alpha\beta}$ is the rotation matrix at site i, $\tilde{S}_i^{\alpha}$ is the coordinate of the rotated spin, relative to rotated coordinates, and λ_i^{α} are the eigenvalues of the matrix $\Delta_i^{\alpha\beta}$. If the matrix elements $\Delta_i^{\alpha\beta}$ have a fixed relationship relative to each other (i.e., independent of lattice site), then the rotation matrix $R_i^{\alpha\beta}$ and the spin coordinate axes will be *independent* of site. The Hamiltonian (4) can then be written as

$$H = -\sum_{\langle ij\rangle} J\tilde{\boldsymbol{S}}_i \cdot \tilde{\boldsymbol{S}}_j - \sum_i \sum_{\alpha=1}^{n} \lambda_i^{\alpha} (\tilde{S}_i^{\alpha})^2. \tag{5}$$

The exchange term is thus *non-random* once again. While the eigenvalues λ_i^{α} will have fixed ratios throughout the system, their values will vary randomly from site to site. Note that if we now shift say the $\tilde{S}^1$ component by M we do not generate a random transverse field. In fact for infinite anisotropy strength, the system will order along the coordinate direction with the largest value of λ_i^{α}. Since these eigendirections are orthogonal to each other, there will be no exchange coupling between spins ordered along different eigendirections. The system then decouples into n independent random Ising models†. The randomness arises because spins ordered along a common eigendirection are scattered randomly about the system. The fraction of such sites should be proportional to $1/n$. Thus, if $1/n$ is greater than the percolation threshold p_c of the lattice in question we would expect ferromagnetism to exist. For example, for $n = 2$, ferromagnetism would exist even in two dimensions for the triangular lattice, and in three dimensions for the simple cubic. The Khmelnitskii fixed point (Khmelnitskii 1975) should describe the continuous phase transition into the low-temperature phase. The order is very non-uniform with spins aligned along the mutually orthogonal eigendirections.

For finite values of the anisotropy we expect the ferromagnetic order to persist below four dimensions, with the ordering becoming progressively more uniform as the strength of the anisotropy is lowered. This supposition has been confirmed for a special case of (5) where λ_i^{α} is zero for all values of α except one value which varies randomly from site to site. This model is the 'cubic' random axis model first studied by Aharony (1975). Mukamel and Grinstein (1982) have shown that near four dimensions this model does indeed exhibit a Khmelnitskii transition at finite values of the anisotropy,

† This statement assumes that the eigenvalues are non-degenerate. Those eigenvalues which are m-fold degenerate will correspond to models with $O(m)$ symmetry.

for *all* values of n. Presumably, percolation considerations are unimportant when the anisotropy is finite.

We have further substantiated our results by constructing a Mermin–Wagner (Mermin and Wagner 1966, Mermin 1967), style 'proof' for $n=2$ utilising the replica trick. Similar 'proofs' have been done for the random p-fold anisotropy models (Schuster 1977, Pelcovits 1979). For the model given by (5) we find the following inequality, characteristic of non-random systems

$$1 \geqslant M^2 k_{\rm B} T \int {\rm d}^d k / \sigma k^2 \tag{6}$$

where σ is finite and proportional to J. This result suggests that long-range ferromagnetic order can exist down to two dimensions. On the other hand, if we consider the more general model where the coefficients $\Delta_i^{\alpha\beta}$ vary randomly with no fixed relationship among coefficients at a single site we find

$$1 \geqslant M^2 k_{\rm B} T \int \frac{{\rm d}^d k}{\sigma k^2} + M^2 F\{C\} \int \frac{{\rm d}^d k}{\sigma^2 k^4} \tag{7}$$

where $F\{C\}$ is some function of the second cumulants C of the distributions of the coefficients $\Delta_i^{\alpha\beta}$, vanishing when $\Delta_i^{\alpha\beta}=0$. The result (7) is similar to that found in the random-field and random-axis model (Schuster 1977, Pelcovits 1979) and suggests that ferromagnetic order is absent below four dimensions even at zero temperature.

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